Power efficient optical OFDM

J. Armstrong and A.J. Lowery

A new technique for using orthogonal frequency division multiplexing (OFDM) in optical systems is presented. Clipped OFDM is derived from a bipolar OFDM waveform by setting the negative values to zero. It has an optical efficiency 8 dB better than DC biased OFDM. If only the odd OFDM subcarriers are modulated, the clipping noise is orthogonal to the wanted signal.

Introduction: Orthogonal frequency division multiplexing (OFDM) allows high-speed data transmission across a dispersive channel, so is used in many new and emerging high-speed wired and wireless communication systems. However, OFDM is not used in commercial optical communication systems. This is because OFDM signals are bipolar, while in optical systems that use intensity modulation (IM), only unipolar signals can be transmitted. A large DC bias is usually added to OFDM signals requiring a high mean optical power with a low modulation depth. This is very inefficient. We show how optically power efficient OFDM signals can be designed. We describe the new technique applied to a baseband signal, but it is also applicable to passband signals.

OFDM signals: In OFDM, signals are transmitted in parallel on a number of subcarriers at different frequencies. Usually quadrature amplitude modulation (QAM) modulates each subcarrier. The transmitter uses an inverse fast Fourier transform (IFFT) to generate a sampled waveform. Let $X(m)$ be the complex number representing the constellation point on the mth subcarrier of a given symbol. Then the baseband time domain samples for that symbol are given by $x(k)$ where

$$x(k) = \frac{1}{N}\sum_{m=0}^{N-1} X(m)\exp\left(\frac{-j2\pi km}{N}\right) \tag{1}$$

and N is the size of the IFFT. In general, $x(k)$ and $X(m)$ are complex. For baseband systems the frequency domain vector $\mathbf{X}$ is constrained to have Hermitian symmetry, so that $x(k)$ is real. Fig. 1*a* shows samples of a typical OFDM baseband symbol and the waveform $x(t)$ which could be generated from them. Fig. 1*b* shows a symbol where only odd subcarriers are used. In this case, for clarity, $N = 32$, although typical values range from 64 to 8192. OFDM signals have a high peak-to-average power ratio. For $N \geq 64$ the central limit theorem applies and the distributions of $x(k)$ and $x(t)$ are approximately Gaussian.

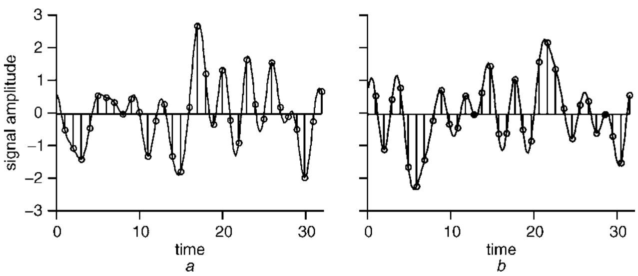

Fig. 1 *OFDM time domain signal*
a All subcarriers modulated
b Only odd subcarriers modulated
——— analogue signal, $x(t)$
—o— signal samples, $x(k)$

Optical OFDM: In the few papers that describe the use of OFDM in optical IM systems [1, 2] a unipolar signal $x_{dc}(t)$ is derived from $x(t)$ by adding a DC bias. In Fig. 2*a* the bias is twice the standard deviation of $x(t)$. For a fixed bias, there will be occasional OFDM symbols with large negative peaks which will be clipped, adding noise to the signal. Because $x_{dc}(t)$ gives the intensity of the optical signal, the average transmitted optical power is approximately equal to the DC bias, which in this example is 2, for an RMS electrical power of unity. Thus this system is very inefficient in terms of optical power.

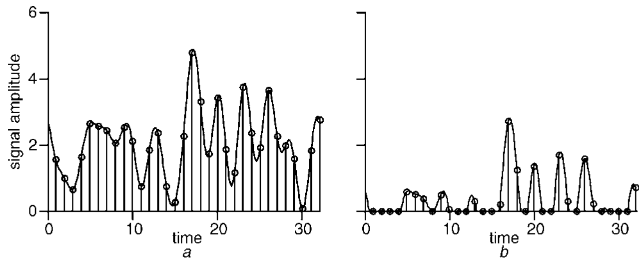

Fig. 2 *Optical signals*
a DC biased OFDM
b Clipped OFDM
——— analogue signal, $x_{dc}(t)$
—o— signal samples, $x_{dc}(k)$

We propose using no bias. In the new scheme the signal $x_c(t)$ shown in Fig. 2*b* would be transmitted. All negative values are forced to zero. We will show that if the subcarrier frequencies used for data transmission are correctly chosen, the data can be retrieved from a signal of this form with little, or for some configurations, no in-band clipping noise.

Analysis of clipping: Clipping is a memoryless nonlinear operation on $x(k)$. As $x(k)$ has a Gaussian distribution, Bussgang's theorem can be applied [3]. Thus

$$x_c(k) = Kx(k) + d(k) \tag{2}$$

where K is a constant and $d(k)$ is a random noise process which is uncorrelated with $x(k)$ so $E[d^*(k)x(k)] = 0$. Thus multiplying (2) by $x^*(k)$ and taking the expectation $E[.]$ gives

$$K = E[x_c(k)x^*(k)]/E[x(k)x^*(k)] \tag{3}$$

$x(k)$ is real so $x^*(k) = x(k)$. Also as $x(k)$ is symmetrically distributed about zero $E[x_c(k)x(k)] = E[x(k)x(k)]/2$ and therefore $K = 1/2$ and $x_c(k) = x(k)/2 + d(k)$. Thus clipping the signal at zero reduces the amplitude of the wanted signal by half, but advantageously it reduces the mean optical power, $E[x_c(k)]$, by much more. $x_c(k)$ is zero with probability 0.5 and has a semi-normal distribution otherwise, so

$$E[x_c(k)] = \left(1/\sqrt{2\pi}\right)\int_0^\infty z\,\exp(-z^2/2)dz = 1/\sqrt{2\pi}_{[j1]} \tag{4}$$

Thus the average optical power of $x_c(k)$ is approximately $1/\sqrt{(2\pi)}$ compared to 2 for DC-biased optical OFDM. The amplitude of each OFDM subcarrier is also reduced by a factor of 0.5. When both the electrical and optical effects of clipping are considered, the electrical amplitude of each subcarrier is increased by a factor of $0.5 \times 2 \times \sqrt{(2\pi)}$ compared with DC-biased OFDM with the same average optical power. Thus the overall improvement in electrical signal-to-noise ratio is $20\ \log_{10}(\sqrt{(2\pi)}) = 7.98$ dB. Clipping also adds an unwanted noise component, $d(k)$. Most of the noise power is in the zeroth subcarrier and is easily separated from the OFDM signal, but some is at other frequencies.

Frequency distribution of clipping noise: The distribution of the energy of $d(k)$ across subcarrier frequencies depends on which subcarriers are used in the unclipped OFDM signal. If only the odd subcarriers of $x(k)$ in a symbol are nonzero then it can be shown that $d(k)$ has components at even frequencies only. Consider the properties of the IFFT and FFT and the relationship between $x(k)$ and $x(k + N/2)$ for odd subcarriers only. The component of $x(k)$ due to the mth subcarrier is given by

$$x(m, k) = (1/N)X(m)\exp(j2\pi km/N) \tag{5}$$

Thus, it is simple to show that for m odd $x(m, k) = -x(m, k + N/2)$. Thus an OFDM symbol consisting of only odd frequencies has the property that $x(k) = -x(k + N/2)$ as shown in Fig. 1*b*. The value of $X(m)$ can be calculated from $x(k)$ using an FFT,

$$X(m) = \sum_{k=0}^{N-1} x(k)\exp\left(\frac{-j2\pi km}{N}\right) \tag{6}$$

To derive the properties of the clipped signal it is useful to modify (6) to give

$$\begin{aligned} X(m) = & \frac{1}{N}\sum_{\substack{k=0\\x(k)>0}}^{N/2-2} x(k)\exp\left(\frac{-j2\pi km}{N}\right) \\ & + x\left(k+\frac{N}{2}\right)\exp\left(\frac{-j2\pi(k+N/2)m}{N}\right) \\ & + \frac{1}{N}\sum_{\substack{k=0\\x(k)<0}}^{N/2-2} x(k)\exp\left(\frac{-j2\pi km}{N}\right) \\ & + x\left(k+\frac{N}{2}\right)\exp\left(\frac{-j2\pi(k+N/2)m}{N}\right) \end{aligned} \tag{7}$$

If only odd subcarriers are nonzero then the second term in each of the summations in (7) is equal to the first term so

$$X(m) = \frac{2}{N}\sum_{\substack{k=0\\x(k)<0}}^{N/2-2} x(k)\exp\left(\frac{-j2\pi km}{N}\right) + \frac{2}{N}\sum_{\substack{k=0\\x(k)<0}}^{N/2-2} x(k)\exp\left(\frac{-j2\pi km}{N}\right) \tag{8}$$

Now consider the case of clipping a signal with only odd frequencies. Then in (7) only the first term in the first summation and the second term in the second summation are nonzero. Thus comparing with (8) and summing over the positive signal values gives

$$X_c(m) = \frac{1}{N}\sum_{k=0}^{N-1} x_c(k)\exp\left(\frac{-j2\pi km}{N}\right) = \frac{X(m)}{2} \tag{9}$$

Thus clipping in this case results in the amplitude of each of the (odd) data carrying subcarriers being *exactly* half of their original value and therefore all of the clipping noise must fall on the even subcarriers. Thus the received signal can be demodulated in exactly the same way as a standard OFDM system, the data is recovered from the odd subcarriers and the even subcarriers are discarded.

This analytical result was confirmed by simulation. Fig. 3 shows the scatter plot for 4QAM, $N=32$ and 20 OFDM symbols with only odd subcarriers carrying data. The constellation points before clipping were $\{\pm 1, \pm j\}$. After clipping the constellation points for the odd subcarriers become as predicted $\{\pm 0.5, \pm 0.5j\}$ the even subcarriers have a noise-like distribution, while the zeroth subcarrier is positive for every symbol but varies slightly from symbol to symbol depending on the data.

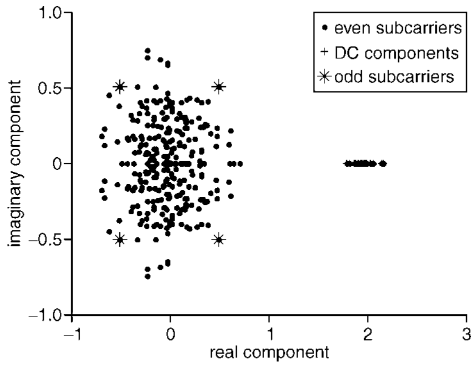

Fig. 3 *Signal constellation after clipping*

Discussion and conclusions: A new technique for using OFDM in IM optical wireless or fibre systems has been described. A conventional bipolar OFDM signal is generated but this is then clipped at zero to generate clipped OFDM. Clipped OFDM has an optical power efficiency approximately 8 dB better than previously described optical OFDM systems. Because the modification from standard OFDM involves quite small changes in the signal format, most of the well established OFDM techniques for equalisation, synchronisation, error coding, etc., can be applied with little modification. OFDM has proved to be a simple and effective solution to signal dispersion in wired and radio applications; clipped OFDM promises to provide a similar, optically power efficient solution in the optical domain. Analytical and simulation results have been presented for the case where only the odd subcarriers of the unclipped OFDM signal are used. In this case, the distortion is orthogonal to the wanted signal and so the technique causes no degradation to the useful signal.

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